Introduction

This is a supplement to help you with three of the most common challenges in understanding and performing statistical hypothesis tests:

1. Translation

"I don't know how to translate the wording of the question into knowing what test I should be performing."

2. Writing down the <u>null (H_0) </u> and <u>alternative (H_A) </u> hypotheses.

"I'm not sure which is the null and which is the alternative and I don't know whether to do a one sided or a two sided p-value calculation."

3. Knowing which **probability distribution calculator** to use.

1. Tips for Translation

BIOSTATS 540 (Introductory Biostatistics) provides an introduction to only a limited number of hypothesis testing scenarios. The sample is either from one sample or two, the data are either paired or independent, and the distribution of the variable is either normal or binomial.

TIP #1 - As you read each statement of the problem, think systematically and identify the following:

	Question	Answer
1	What is the design?	
	ONE group	
	PAIRED	
	TWO independent groups	
2	What type of outcome data is this?	
	CONTINUOUS(normally distributed) with:	
	* Population variance(s) known	
	* Population variance(s) NOT known	
	DISCRETE (binomial outcome data)	
3	What is the focus ?	
	MEAN (one or two)	
	VARIANCE (one or two)	
	BINOMIAL EVENT PROB (one or two)	
4	One sided or Two sided?	

[&]quot;I can't keep track of this."

2. Writing down the null (H_0) and alternative (H_A) hypotheses.

Remember. Most of the time (there are exceptions, of course, but we'll get to these), the investigator is seeking to advance the alternative hypothesis (e.g., - the new treatment is great). The null hypothesis is the "nothing is going on" hypothesis (as the term "null" suggests!)

TIP #2 - Write down the **alternative hypothesis** <u>first</u>. Take care to indicate whether this is one sided or two sided (see tip #3 next).

TIP #3 - To discern one versus two sided, re-read each statement of the problem, looking for and then highlighting, the words that tell you whether the test to do should be one sided or two sided.

Words that Suggest Test is ONE sided	Words that Suggest Test is TWO sided
less than	different
reduced	has changed
greater	
improved	

3. Knowing Which Probability Calculator to Use.

In a nutshell: If the test is of a mean or difference of means, the correct calculator is the Normal(0,1) or Student t. If the test is of a variance, the correct calculator is chi square. If the test is of a ratio of variances, the correct calculator is the F distribution. If you go back to the table of contents, you could write these in. Here is what you would get, highlighted in **bold green italics**.

1 Group, Outcome Continuous and Distributed Normal

- 6. Test for μ , σ^2 Known calculator: Normal(0,1)
- 7. Test for μ , σ^2 Known Critical Region Approach calculator: Normal(0,1)
- 8. Test for μ , σ^2 Unknown calculator: Student t
- 9. Test for σ^2 calculator: Chi Square

Paired Data, Outcome Continuous and Distributed Normal

10. Test for μ_{DIFFERENCE} – Paired Data Setting CHOOSE ONE

calculator: Normal(0,1) if population variance known calculator: Student t if population variance is NOT known

2 Independent Groups, Outcome Continuous and Distributed Normal

11. Test for $[\mu_1 - \mu_2]$ CHOOSE ONE

calculator: Normal(0,1) if population variances are known calculator: Student t if population variances are NOT known

- 12. Test for Equality of Two Variances (σ_1^2/σ_2^2) calculator: F distribution
- 1 Group, Outcome Discrete and Distributed Binomial
 - 13. Test for Proportion π CHOOSE ONE
 - 13.1 Exact Test calculator: Exact Binomial
 - 13.2 Normal Approximation Test calculator: Normal(0,1).
- 2 Independent Groups, Outcome Discrete and Distributed Binomial
 - **14.** Test for $[\pi_1 \pi_2] = 0$ calculator: Normal(0,1)

Examples

Example #1

The personnel department of a large company would like to determine if the amount of time it takes for employees to arrive at work. A random sample of 12 employees is selected and the time in minutes is recorded, with the following results:

15 30 50 60 25 65 45 90 75 50 50 20

At the .01 level of significance, is there evidence that the average travel time of employees is less than 60 minutes?

Solution:

	Question	Answer
1	What is the design ?	ONE group
	ONE group	
	PAIRED	
	TWO independent groups	
2	What kind of data is this?	CONTINUOUS, normal with
	CONTINUOUS(normally distributed) with:	variance NOT known.
	* Population variance(s) known	
	* Population variance(s) NOT known	
	DISCRETE (binomial outcome data)	
3	What is the focus ?	MEAN of one population
	MEAN (one or two)	
	VARIANCE (one or two)	
	BINOMIAL EVENT PROB (one or two)	
4	One sided or Two sided?	ONE sided because of wording
		<mark>less than</mark> 60 minutes

Correct Test to Use: One Sample t test for μ , where σ^2 is not known.

Null: $\mu = 60$

Alternative: $\mu < 60$, one sided

An auditor for the Department of Energy wishes to study the price of unleaded gasoline per gallon in NY city. A random sample of 50 gas stations was selected with the following results: $\bar{X} = \$1.36$ and S = \$0.07 Is there evidence that the average price of unleaded gasoline is different from \$1.30 per gallon?

Solution:

	Question	Answer
1	What is the design ?	ONE group
	ONE group	
	PAIRED	
	TWO independent groups	
2	What kind of data is this?	CONTINUOUS, normal with
	CONTINUOUS(normally distributed) with:	variance NOT known.
	* Population variance(s) known	
	* Population variance(s) NOT known	
	DISCRETE (binomial outcome data)	
3	What is the focus ?	MEAN of one population
	MEAN (one or two)	
	VARIANCE (one or two)	
	BINOMIAL EVENT PROB (one or two)	
4	One sided or Two sided?	TWO sided because of wording
		different from \$1.30 per gallon

Correct Test to Use: One Sample t test for μ , where σ^2 is not known.

Null: $\mu = \$1.30$

Alternative: $\mu \neq 1.30 two sided

A consumer reporting agency wished to determine whether an "unknown brand" calculator sells at a lower price than the "famous brand" calculator of the same type. A random sample of eight stores was selected and the prices (at the stores) of each of the two calculators were recorded with the following results.

Store	Unknown Brand	Famous Brand
1	10	11
2	8	11
3	7	10
4	9	12
5	11	11
6	10	13
7	9	12
8	8	10

At the .01 level of significance, is there evidence that the unknown brand sells for a lower price?

Solution:

	Question	Answer
1	What is the design ?	PAIRED
	ONE group	
	PAIRED	
	TWO independent groups	
2	What kind of data is this?	CONTINUOUS, normal with
	CONTINUOUS(normally distributed) with:	variance NOT known.
	* Population variance(s) known	
	* Population variance(s) NOT known	
	DISCRETE (binomial outcome data)	
3	What is the focus ?	MEAN of one population
	MEAN (one or two)	
	VARIANCE (one or two)	
	BINOMIAL EVENT PROB (one or two)	
4	One sided or Two sided?	ONE sided because of wording
		sells for a lower price

Correct Test to Use: Paired data one Sample t test for $\mu_{DIFFERENCE}$ where $\sigma^2_{DIFFERENCE}$

is not known.

Null: $\mu_{DIFFERENCE} = 0$ for difference defined [Unknown – Famous]

Alternative: $\mu_{\text{DIFFERENCE}} < 0$ one-sided

Olmstead [1953] conducted a study of cases with convulsive disorders. Among the cases there were 82 males and 118 females. At the 5% significance level, test the hypothesis that a case is equally likely to be of either sex.

Solution:

	Question	Answer
1	What is the design ?	ONE group
	ONE group	
	PAIRED	
	TWO independent groups	
2	What kind of data is this?	DISCRETE, Binomial.
	CONTINUOUS(normally distributed) with:	
	* Population variance(s) known	
	* Population variance(s) NOT known	
	DISCRETE (binomial outcome data)	
3	What is the focus ?	Binomial event probability
	MEAN (one or two)	
	VARIANCE (one or two)	
	BINOMIAL EVENT PROB (one or two)	
4	One sided or Two sided?	TWO sided because of wording
		<mark>equally likely</mark>

Correct Test to Use: One Sample Test for Proportion π .

Null: $\pi = 0.50$

Alternative: $\pi \neq 0.50$ two sided

Holtzman et al [1975] conducted a study of a diet designed to reduce the phenylalanine level in children with phenylketonuria (PKU). After obtaining informed consent, eligible children of 4 years of age were randomly divided into two groups. Children in one group received the new, experimental, diet. Children in the other group followed their usual diet. Study investigators measured the phenylalanine levels (mg/dl) at the end of the study with the following results.

	Experimental Diet	Usual Diet
Number in group, n	4	5
Mean phenylalanine (mg/dl)	16.7	26.9
Standard deviation	7.3	4.1

Solution:

	Question	Answer
1	What is the design ?	TWO independent groups
	ONE group	
	PAIRED	
	TWO independent groups	
2	What kind of data is this?	CONTINUOUS, normal with
	CONTINUOUS(normally distributed) with:	variances NOT known.
	* Population variance(s) known	
	* Population variance(s) NOT known	
	DISCRETE (binomial outcome data)	
3	What is the focus ?	MEANS of two populations
	MEAN (one or two)	
	VARIANCE (one or two)	
	BINOMIAL EVENT PROB (one or two)	
4	One sided or Two sided?	ONE sided because investigators
		hope that diet will reduce
		phenylalanince levels

Correct Test to Use: Two Sample t test for equality of $\mu_{EXPERIMENTAL}$ and μ_{USUAL} ,

where $\sigma^2_{EXPERIMENTAL}$ and σ^2_{USUAL} are not known.

Null: $\mu_{\text{EXPERIMENTAL}} = \mu_{\text{USUAL}}$ which is the same as $\left[\mu_{\text{USUAL}} - \mu_{\text{EXPERIMENTAL}}\right] = 0$

Alternative: $\mu_{\text{EXPERIMENTAL}} < \mu_{\text{USUAL}}$ or $\left[\mu_{\text{USUAL}} - \mu_{\text{EXPERIMENTAL}}\right] > 0$ one sided